moderately low strength, and in some cases, limited availability.

VII. Conclusions

1) The development of a vapor plating apparatus has led to the first successful synthesis of continuous pyrolytic graphite fibers.

2) These continuous filaments represent an important new physical form of graphite. As such, they bridge the limited availability gap between very high strength graphitic whiskers and relatively low strength pyrolyzed rayon cloths.

3) The pyrolysis of certain substituted alkane hydrocarbons produce more finely grained, higher strength, and more uniform pyrolytic graphite than other conventional hydrocarbon compounds.

4) The graphitic fibers are a first generation materials. Further research is required to improve the unique properties of these fibers and to evaluate them in composites.

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Investigation of the Charge-to-Mass Ratio of Electrically Sprayed Liquid Particles

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An analysis of the charge-to-mass ratios (specific charges) of particles generated by the electrical atomization process is presented. The study involves the surface energy of the dispersed system of particles, the effects of space charge on the source, and the effects of conductivity on the atomization process. Experimental data are presented in support of these theories. Further, a colloidal suspension in glycerine is experimentally shown to produce particles of high specific charge (400 coul/kg) when electrically dispersed under appropriate conditions.

I. Introduction

THE electrical spraying of liquids or the electrical atomization of liquids is, basically, a process by which a liquid volume is broken up into small, charged particles as a result of electrostatic pressures overcoming surface tension pressures. According to Drozin,[1] the electrical atomization of liquids was first observed by George Bose in 1745. Rayleigh[2] determined an instability criterion for charged, liquid droplets in 1882. In 1917, Zeleny[3] established a criterion for the instability of an electrified liquid at the end of a capillary tube. Drozin,[1] in 1955, considered the electrical forces causing instability of liquid surfaces as arising from dielectric stresses in the liquid. He does not explain, however, how charged particles can form as the result of dielectric stresses.

In 1952, Thomas[4] advanced the idea of the electrical acceleration of charged colloidal particles to produce thrust for space vehicles. Particles having specific-charges in the range 200 to 10^4 coul/kg would be very useful for propulsion purposes and would tend to bridge the mission capability gap between arcjet thrusters and cesium ion thrusters.[5] Some of the first research on electrical spraying of liquids in application to space-vehicle propulsion was reported by Shultz and Branson[6] and by Hendricks.[7] Other contributions have been submitted by Krohn[8] and Cohen.[9] These studies established the statistical nature of the charge and mass distributions of emitted particles and showed the general shape of current distributions of specific charge for some liquids.

This paper presents a study of the fundamental mechanisms involved in the electrical dispersion of liquids from the tip of capillary tubes. In this process, charged colloidal-size particles are generated by allowing a liquid to flow through a small, metal capillary tube maintained at a high electric potential. Theoretical treatments and experimental results are presented concerning the following aspects of the problem: 1) surface energy minimization of the dispersed system of particles produced from unstable liquids that disperse into

particles having specific charges that are dependent on their radius and on the surface tension of the liquid; 2) Poisson's equation and the influence of space charge on the specific charge of emitted particles; and 3) the influence of the conductivity of the liquid on the generation of the particles. In addition, data on recent experiments in the electrical atomization of solutions containing colloidal suspensions of solid particles are presented.

II. Theoretical Considerations

The specific charge for a droplet oscillating in the lowest vibrational mode can be derived from Rayleigh's work and is found to be

$$q/m = 6[(\epsilon_0\gamma)^{1/2}/\rho r^{3/2}] \tag{1}$$

This equation represents the maximum specific charge that a droplet of a given liquid and radius can have. A formula that indicates the most likely (or average) specific charge of droplets emitted from an unstable volume V having total charge Q can be obtained by considering the energy minimization of droplets thus formed. This technique has been employed by Vonnegut.[10] Suppose the unstable liquid were to break up into N equal size particles having radius r and charge q. The total energy (neglecting interparticle energies) of the system is given by

$$W = N\left(\frac{3\gamma V}{rN} + \frac{Q^2 V}{2\epsilon_0 N(4\gamma r^2)^2}\right)$$
$$= \frac{3\gamma V}{r} + \frac{r^2 Q^2}{18\epsilon_0 V} \tag{2}$$

where γ is the surfacet ension of the liquid. Setting $dW/dr = 0$ for the energy extremum, the radius of the emitted droplets is found to be

$$r = (27\epsilon_0\gamma V^2/Q^2)^{1/3} \tag{3}$$

or, expressing the result in terms of the total mass M,

$$Q = M[(27\epsilon_0\gamma)^{1/2}/\rho r^{3/2}] \tag{4}$$

Since the specific charge of each of the dispersed droplets (q/m) is the same as the initial charge-to-mass ratio (Q/M), we have

$$q/m = (27\epsilon_0\gamma)^{1/2}/\rho r^{3/2} \tag{5}$$

Thus, the radius and specific charge of the emitted droplets is determined by the initial, unstable volume of liquid having charge Q. In fact, if a capacitance C is associated with the source geometry, then the maximum charge on the source surface is $Q = CU_0$, and we have

$$q/m = CU_0/\rho V \tag{6}$$

where U_0 is the electrical potential and ρ is the density of the liquid.

Equation (6) indicates that the specific charge should increase linearly with the accelerating voltage. As is shown in the next section, significantly different results are obtained when space charge in the vicinity of the source is considered.

Space Charge

Consider the problem of particle emission from a spherical source of radius r_0 into a radial field produced by a spherical accelerating electrode concentric with the source and having radius r_d. Poisson's equation for such a geometry is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU}{dr}\right) = -\frac{\rho_c}{\epsilon_0} \tag{7}$$

where ρ_c is the charge density. For the present problem, Eq. (7) is subject to the following boundary conditions: 1) $r = r_0$, $U = 0$, $dU/dr = E_0$, $I_0 = 4\gamma r_0^2 J_0$; 2) $r = r_d$, $U = -U_d$. J_0 is the current density at the surface $r = r_0$, and $|U_d| = |U_0|$. These boundary conditions assume an elec-

tric field at the source in contrast to the zero field for space-charge limited conditions. Equation (7) may be expressed in the form

$$\frac{d^2U}{dr^2} + \frac{2}{r}\frac{dU}{dr} = -\frac{J(r)K}{(-U)^{1/2}} \tag{8}$$

where $J(r)$ is the current density at radius r, and $K = 1/\epsilon_0 2(q/m)^{1/2}$. A direct solution of this quasi-linear equation in closed form would be difficult if not impossible. For large r, the second term on the left may be neglected and $J(r)$ may be assumed constant.[11] Thus the equation reduces to

$$d^2U/dr^2 = -JK/(-U)^{1/2} \tag{9}$$

Integrating this equation, we have

$$(dU/dr)^2 = 4JK(-U)^{1/2} + C_1' = 4JK[(-U)^{1/2} + C_1] \tag{10}$$

Applying the first boundary condition results in

$$C_1 = E_0^2/4J_0K \tag{11}$$

to a good approximation. Substituting the expression $I_0/4\gamma r_0^2$ for the current density J_0, the integral equation for Eq. (10) becomes

$$\int \frac{dU}{(-U)^{1/2} + C_1)^{1/2}} = -\left(\frac{KI_0}{\pi}\right)^{1/2}\int \frac{dr}{r} \tag{12}$$

Performing the indicated integrations and applying the boundary conditions, we have

$$B(KI_0/4\pi)^{1/2} = \tfrac{2}{3}[(-U_0)^{1/2} + C_1]^{3/2} - 2C_1[(-U_0)^{1/2} + C_1]^{1/2} + \tfrac{4}{3}C_1^{3/2} \tag{13}$$

where B is assumed to be given by a geometric series[11] of $u = \ln(r_d/r_0)$. This series is generally such that B may be taken as 1 when $r_d/r_0 \gg 1$, which will be assumed for this problem.

Since C_1 is a function of specific charge, further approximations must be made in order to solve for the specific charge in terms of the other quantities. The following two cases will be considered: case I, $C_1 \ll (U_0)^{1/2}$; case II, $C_1 \gg (U_0)^{1/2}$.

Case I: The assumption that C_1 is negligible compared to $(U_0)^{1/2}$ implies that the electric field at the source is very small. Using this assumption, Eq. (13) reduces to

$$(KI_0/4\pi)^{1/2} = \tfrac{2}{3}U_0^{3/4} - 2C_1U_0^{1/4} + \tfrac{4}{3}C_1^{3/2} \tag{14}$$

Equation (14) may be solved for the specific charge by using the relation $I_0 = (q/m)\dot{M}$, where $\dot{M}$ is the mass flow rate. The result is, approximately,

$$\frac{q}{m} = \frac{128\pi^2\epsilon_0^2U_0^3\{1 + [1 - (27\mathcal{E}_0^2/2)]^{1/2}\}^2}{81\,\dot{M}^2} \tag{15}$$

where $\mathcal{E}_0 = E_0r_0/U_0$. This equation reduces to Child's law for $\mathcal{E}_0 = 0$ and is valid for $\mathcal{E}_0^2 \leq \tfrac{2}{27}$.

Case II: When it is assumed that $C_1 \gg (U_0)^{1/2}$, Eq. (13) may be solved by expanding and collecting the fractional exponent terms. Using the first four terms of each of these expansions, the result is

$$(KI_0/4\pi)^{1/2} = \tfrac{1}{2}(U_0/C_1^{1/2}) - \tfrac{1}{6}(U_0^{3/2}/C_4^{3/2}) \tag{16}$$

Solving this equation for the specific charge, we have

$$q/m = 18\pi^2\epsilon_0^2U_0^3\mathcal{E}^4[(1 - \mathcal{E}_0)^2/\dot{M}^2] \tag{17}$$

This equation, it must be emphasized, is valid only for $\mathcal{E}_0$ approximately equal to 1.

Equations (15) and (17) become more applicable to the problem of spraying from the tip of a capillary tube if they are multiplied by the square of the solid angle of dispersion ratio $\Omega^2/(4\pi)^2$. Thus Eqs. (15) and (17) become

$$q/m = 8\epsilon_0^2U_0^3\Omega^2\frac{\{1 + [1 - (27\mathcal{E}_0^2/2)]^{1/2}\}^2}{81\,\dot{M}^2} \tag{18}$$

and

$$q/m = 9\epsilon_0^2U_0^3\mathcal{E}_0^4\Omega^2[(1 - \mathcal{E}_0)^2/8\dot{M}^2] \tag{19}$$
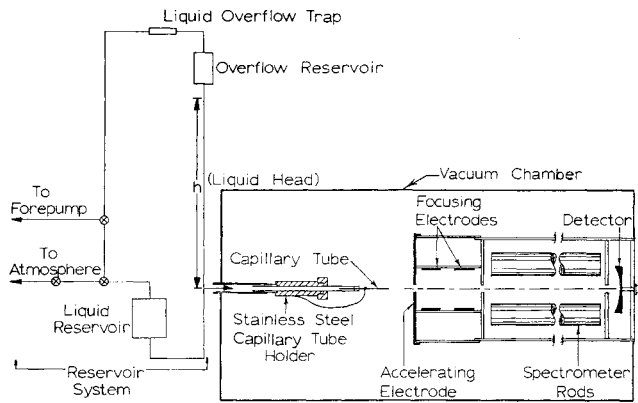
respectively.

Fig. 1 Schematic diagram of the experimental apparatus used to generate and measure charged, liquid particles.

Thus, with no space charge, the charge-to-mass ratio can be expected to increase linearly with the accelerating voltage whereas with space charge, the specific charge will increase almost as the cube of the accelerating voltage. Since the electric field ratio $\mathcal{E}_0$ can be expected to change with voltage, the specific charge is proportional to the cube of the accelerating potential only for complete space-charge limiting.

Conductivity

Thus far, the conductivity of the liquid has not been taken into account. In fact, it has been tacitly assumed that the liquid surface assumes an equipotential instantaneously and that this equipotential is maintained constant during the emission of particles. However, the finite conductivity of the liquid will be found to have an important influence on the electrical dispersion process.

One approach to the conductivity problem is to consider a current I_0 flowing from a capillary tube as a result of the emission of liquid particles when a voltage U_a is applied to the tube. Because of the finite conductivity of the liquid, a voltage drop will appear between the capillary tube and the point of droplet emission at the liquid tip. As a consequence, the voltage U_0 at the emitting tip of the liquid meniscus is given by

$$U_0 = U_a - [I_0 R_v R_s / (R_v + R_s)] \qquad (20)$$

where I_0 is the total current, and R_v and R_s are the volume and surface resistance, respectively, of the liquid protruding from the capillary tube.

Approximate equations for the volume and surface resistances can be obtained by assuming that the liquid meniscus forms a hemisphere at the tip of the capillary tube and that a jet of radius r_0 extends from its extremity. The ratio of the radius of the jet to the radius of the capillary tube r_a will be denoted by δ, i.e., $r_0 = \delta r_a$.
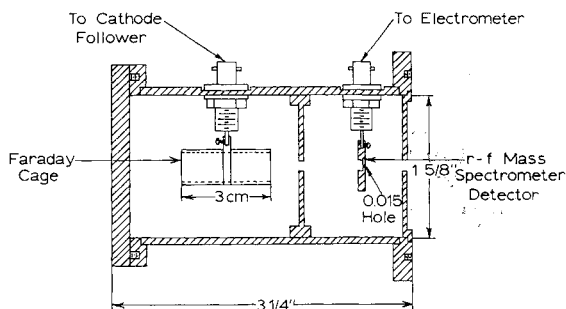

Fig. 2 Schematic diagram of the Faraday cage detector used to measure individual particles resolved by the quadrupole mass spectrometer.

Assuming that the capillary tube approximates a solid conductor with a flat tip, the volume resistance of the liquid between the tube and the jet may be derived using the approximate resistance formula

$$R_v = \frac{1}{\sigma_v} \int \frac{dz}{A} \qquad (21)$$

where z is the axis of symmetry and A is the cross-sectional area of the incremental length dz. Performing the integration, we have

$$R_v = \frac{1}{2\pi\sigma_v r_a} \ln \left| \frac{1 + (1 - \delta^2)^{1/2}}{1 - (1 - \delta^2)^{1/2}} \right| \qquad (22)$$

or, for $\delta^2 \ll 1$,

$$R_v = \frac{1}{2\pi\sigma_v r_a} \ln \left(\frac{4}{\delta^2} \right) \qquad (23)$$

where σ_v is the volume conductivity.

In a similar fashion, the approximate surface resistance may be found using the formula

$$R_{s3} = \frac{1}{\sigma_s} \int \frac{dz}{L} \qquad (24)$$

where σ_s is the surface conductivity and L is the circumference of the liquid surface at distance z. Integration of this equation gives

$$R_s = \frac{1}{2\pi\sigma_s} \sin^{-1}(1 - \delta^2)^{1/2} \qquad (25)$$

or, for $\delta^2 \ll 1$,

$$R_s = \frac{1}{2\pi\sigma_s} \sin^{-1}[1 - \frac{\delta^2}{2}] \qquad (26)$$

The liquid resistance $R = R_s R_v / (R_s + R_v)$ together with the total capacitance C of the dispersion system and the discharge nature of the liquid make up the essential elements of a relaxation oscillator. If the resistance is considered in series with a parallel combination of the capacitance and the discharge device, the period would be given by

$$P = RC \ln(U_a - U_2)(U_a - U_1) \qquad (27)$$

where U_1 is the minimum spraying potential necessary to initiate instability and U_2 is the potential at which emission ceases. In this model, R and C were assumed independent of time although, ordinarily, they are functions of time since they vary with the volume of liquid at the capillary tip and, therefore, with the mass-flow rate to and from the tip. If the mass at the capillary tip is assumed constant during charge build-up, the average specific charge over the period (assuming no space-charge effects) is given by

$$\left(\frac{q}{m} \right)_{\mathrm{av}} = \frac{CU_a}{\rho V} \left[1 - \frac{RC}{P} \left(1 - \frac{U_2}{U_a} \right) \left(1 - \exp\left\{ \frac{-T}{RC} \right\} \right) \right] \qquad (28)$$

Further, when $U_a \gg U_1$ and U_2, the period approaches zero and Eq. (28) approaches

$$(q/m)_{\mathrm{av}} = CU_2/\rho V = CU_2/\rho V \qquad (29)$$

which is Eq. (6).

III. Experimental Apparatus

The apparatus used in the electrical spraying experiments included a high vacuum chamber, a liquid reservoir and particle source assembly, a quadrupole mass spectrometer, and a Faraday cage. Measurements of the conductivity, viscosity, surface tension, and dielectric constant of each liquid sprayed were made in separate experiments.
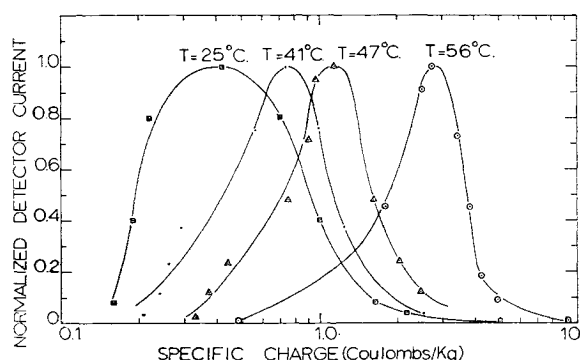
Fig. 3 Current distributions of specific charge for glycerine operating at different temperatures. The operating conditions were: accelerating voltage = 4.1 kv d.c., capillary o.d. = 203 μ (8 mils), liquid reservoir pressure = 64 cm of glycerine.

A schematic diagram of the basic apparatus used is shown in Fig. 1. The reservoir system was used to feed the liquid to the capillary tip and to control the mass-flow rate. In this scheme, the mass-flow rate of liquid through the source tube was directly proportional to the hydrostatic pressure in the reservoir. The accelerating voltage was applied across the capillary tube and the accelerating electrode. The particles passing through the accelerating electrode were focused and then entered the quadratic field of the quadrupole mass spectrometer.

The quadrupole mass spectrometer is a device that separates particles according to their specific charge (q/m) by transverse acceleration of the particles in a time-dependent quadratic electric field. This device, invented by Paul and Steinwedel,[12] was found to be very useful for the study of heavy charged particle distributions. The advantage of this spectrometer for this study lies principally in the fact that only one device is needed to cover a broad range of specific charges. The spectrometer used in these experiments was capable of resolving particles from 5×10^{-3} to 10^7 coul/kg. Other advantages of this spectrometer as compared to other devices include the elimination of cumbersome magnets, ease of construction and installation, and low over-all cost.

The Faraday cage detector shown in Fig. 2 was occasionally used in place of the detector shown in Fig. 1. The small hole in the spectrometer detector allowed only a small fraction of the resolved beam to pass into the Faraday cage. This allowed analysis of single particles by which their charge, mass, size, and velocity could be determined. The pressure in the vacuum chamber was generally maintained at 5×10^{-6} torr.

IV. Results and Discussions

Temperature

Current distributions of specific charge for glycerine operating at different temperatures are shown in Fig. 3. In these experiments, only the liquid exposed at the end of the capillary tube was heated. Thus, the mass-flow rate remained constant. The nonlinear increase in q/m with temperature is either due to a nonlinear reduction in surface conductivity (volume conductivity was known to vary linearly with temperature), or to a nonlinear increase in the vapor pressure, or both.

Space Charge

Typical distributions of Octoil-S operating at different accelerating voltages are illustrated in Fig. 4. Figures 5 and 6 are plots of the peaks of the distributions (average specific charges) for different accelerating potentials and
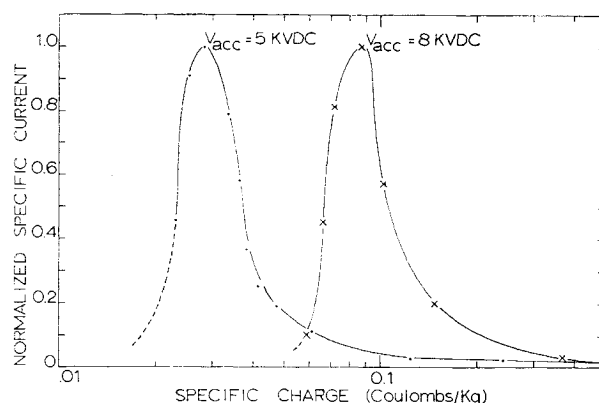
Fig. 4 Current distributions of specific charge for Octoil-S operating at different potentials. The mass flow rate was 2.3×10^{-7} kg/sec and the capillary o.d. was 203 μ (8 mils). V_{acc} is the potential applied to the capillary tube. These curves have been corrected for the bandwidth of the spectrometer and, thus, are expressed in terms of specific current.

flow rates, respectively. The nonlinear increase in q/m with accelerating voltage in Fig. 5 indicates space charge effects. These curves show an increase of q/m with powers of voltage between 1 and 3, which are the limits indicated by Eqs. (6, 18, and 19). Figure 6 shows a general decrease in specific charge with increasing liquid pressures. However, the KOH-Glycerine curve shows a tendency to flatten out at low hydrostatic pressures. This phenomenon was found to be due to liquid pulsations at the capillary tip.

In order to study these pulsations, the Faraday cage arrangement shown in Fig. 2 was employed. Oscilloscope traces were taken of the output of a high-impedance-input cathode follower connected to the Faraday cage. The pulsations are shown in Fig. 7, where the change in the period with increasing mass-flow rate is clearly indicated. The period of the pulsations seemed to change rapidly with little change in specific charge for low hydrostatic pressures. At higher
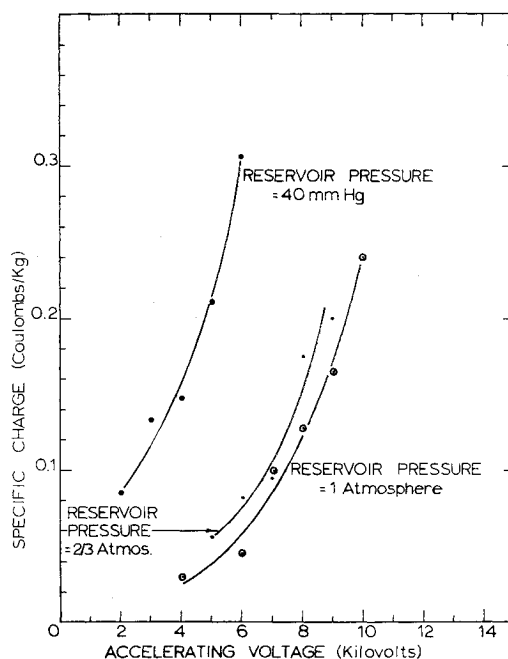
Fig. 5 Plots of average specific charge as a function of accelerating voltage for different reservoir pressures (reservoir pressure is proportional to mass-flow rate). The liquid used was Octoil-S.
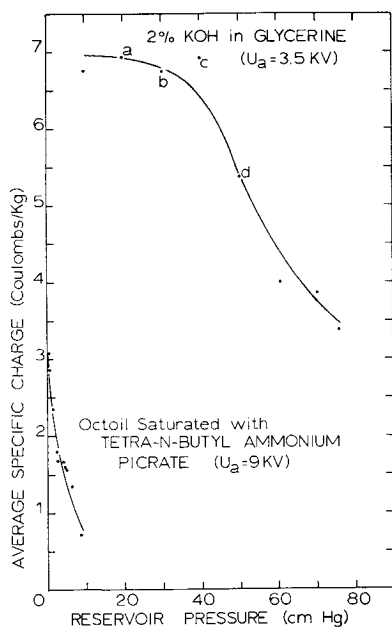
Fig. 6 Plots of average specific charge as a function of reservoir pressure for two different liquids. Reservoir pressure is directly proportional to mass-flow rate.

reservoir pressures, the period approached a constant, allowing the usual decrease in q/m with pressure.

Energy Minimization

The Faraday cage was also used to measure the velocity and charge of particles. The particles measured were a small fraction of those particles passing the mass spectrometer when the latter was tuned to the peak of the distributions. Results of these measurements are presented in Fig. 8. Of significance is the fact that particles having the same specific charge have approximately the same total charge and, therefore, have the same size. The traces that show little decay represent particles hitting the Faraday cage wall, giving rise to potentials that decay with the time constant of the cathode follower input network. The charge was measured from the height and the velocity from the width of the pulse.

Using the radii found from the Faraday cage measurements in the following empirical formula

$$q/m = (18\epsilon_0\gamma)^{1/2}/2\rho r^{3/2} \qquad (30)$$

the specific charges were calculated. The results of these calculations are compared with the measured values in Table

a) Period = 54 msec;
$\dot{m} = 1.7 \times 10^{-9}$ kg/sec.

b) Period = 42 msec;
$\dot{m} = 2.5 \times 10^{-9}$ kg/sec

c) Period = 36 msec;
$\dot{m} = 3.5 \times 10^{-9}$ kg/sec

d) Period = 32 msec;
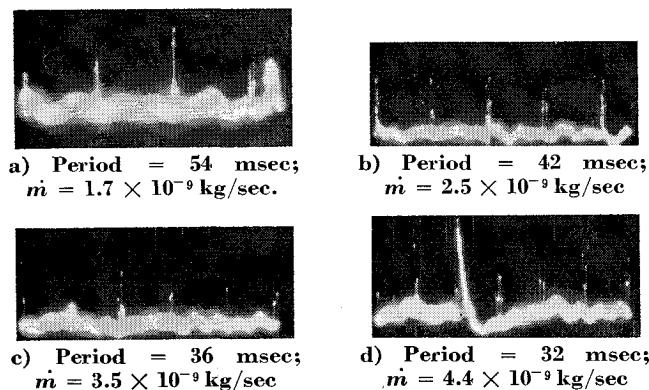$\dot{m} = 4.4 \times 10^{-9}$ kg/sec

Fig. 7 Oscilloscope traces showing the change of period with mass-flow rates of the particle-emission pulsations. The accelerating voltage was 3.5 kv d.c. cf. Fig. 6 for corresponding points on KOH-Glycerine curve.

1. This empirical formula is based on Eq. (5) and has the same form.

High Charge-to-Mass Ratios

Current distributions of specific charge for pure glycerine and for a solution of glycerine containing carbon particles are shown in Fig. 9. The two distributions were obtained under the same conditions (accelerating voltage, hydrostatic pressure, tube size, etc.) The average size of the carbon particles was 12 μ in diameter with a range in size of 2 to 35 μ. A large tube was used to allow the jagged-surfaced particles to pass freely through the tube bore. As shown, the peak of the distribution for the carbon-glycerine solution was 200 coul/kg, whereas that for pure glycerine was about 20 coul/kg.

It was thought that particles having even higher specific charges than 200 coul/kg could be obtained using a true colloidal suspension of smaller particles. A solution of glycerine containing 2% by weight of silicon dioxide particles having a mean diameter of 0.008 μ was subsequently tried. This material, having a very small variance in size, has an unusually large surface area per unit weight (approximately 200 m²/g) and is known for its lack of mutual binding forces and its consequential high propensity to disperse easily in most organic media. Using an 812-μ (32-mil)-o.d. tube, a hydrostatic pressure of about 16 cm of solution, and an accelerating voltage of 17.2 kv, a narrow distribution was found which peaked at 300 coul/kg. A solution containing 3% of silicon dioxide produced two peaks in the current distribution which were located at 405 and 7.9 coul/kg. The current corresponding to the latter was about $\frac{1}{2}$ that of the former.

The volume conductivity of the 3% silicon dioxide in glycerine solution was found to be 2.6×10^{-8} mho/cm whereas the conductivity of bulk-pure glycerine was 5.0×10^{-7} mho/cm. The large surface area of the particles held in suspension allows a great amount of surface activity and this is thought to enhance the specific charge of emitted particles. It is known, for instance, that a small amount of these silicon dioxide particles in water reduces the ph of water considerably.

Negative particles were produced using this solution by applying a negative accelerating voltage to the tube. This experiment produced particles having specific charges of only 3.1 coul/kg. The effect of silicon dioxide on the generation of negative particles might be enhanced (in terms of high specific charges) by adding some alkali metal hydroxides to the solution.

Poisson's Equation and Specific Charge

If the shape of the liquid at the point of instability is assumed to approximate a hyperboloid of revolution, the electric field at the tip is given by[13]

$$E_m = (2U_0/r_0) \ln(4d/r_0) \qquad (31)$$

a) $q_{av} = 7.1 \times 10^{-14}$ coul;
$V_{av} = 33.4$ m/sec

b) $aq_{av} = 4 \times 10^{-14}$ coul;
$V_{av} = 2.72$ m/sec

c) $q_{av} = 2.1 \times 10^{-14}$ coul;
$V_{av} = 33.4$ m/sec

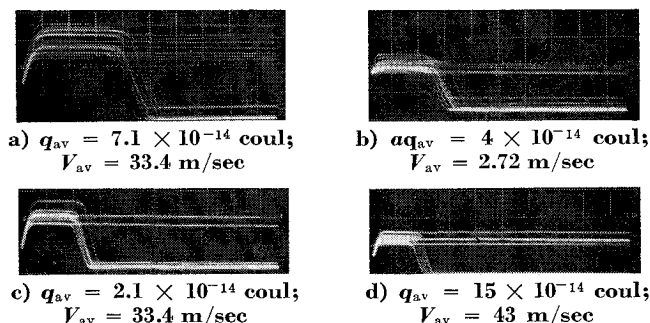d) $q_{av} = 15 \times 10^{-14}$ coul;
$V_{av} = 43$ m/sec

Fig. 8 Oscilloscope traces indicating uniform charges (pulse amplitudes) and uniform velocities (pulse widths) of particles having uniform charge-to-mass ratios. The liquid used in these experiments was Narcoil-40.

Table 1 Comparison of specific-charge measurements and calculations

Faraday cage	0.057	0.12	0.18	0.29	2.50	1.57
Eq. (28)	0.063	0.118	0.20	0.28	2.62	1.78

where

U_0 = potential at the tip

d = distance from the tip to the accelerating plate

r_0 = radius of curvature at the tip

Since E_m is the maximum possible field for this configuration, a correction must be incorporated in Eqs. (18) and (19) to account for the fact that U_0/r_0 is not the maximum. With this correction factor, these equations are assumed to take on the form

$$q/m = 8\epsilon_0^2 U_0^3 K^2 \Omega^2 \frac{\{1 + [1 - (27\epsilon_0^2/2]^{1/2}\}^2}{81 \dot{M}^2} \quad (32)$$

and

$$q/m = 9\epsilon_0^2 K^6 U_0^3 \epsilon_0^4 \Omega^2 [(1 - \epsilon_0)^2/8\dot{M}^2] \quad (33)$$

respectively, where $K = 2/\ln (rd/r_0)$. The solid angle Ω is related to the dispersion angle by $\Omega = 2\pi(1 - \cos\theta)$.

Examples

Consider an example for which the experimental data was as follows; $\dot{M} = 1.2 \times 10^{-10}$ kg/sec, $U_0 = 3.1$ kv, $r_0 = 6.7$ μ (r_0 was taken equal to the average size of the particles), and $d = 2$ cm. The dispersion angle for this low operating voltage was very small and is assumed to be $\theta = 5°$. From other data, $\epsilon_0 = 0.94$ seemed to be a reasonable value for the electric-field ratio. Substituting these values into Eq. (33), i.e., for the equation holding for ϵ_0 near 1, the calculated specific charge is $q/m = 0.034$ coul/kg. The value measured with the Faraday cage was $q/m = 0.057$ coul/kg.

Equation (33) does not hold for the production of specific charges in the 400 coul/kg range. The electric field ratio appears to be limited to values which are too high. As an example, consider the results obtained using 3% silicon dioxide in glycerine. In this experiment, the measured variables were: $\dot{M} = 4.1 \times 10^{-8}$ kg/sec, $d = 2$ cm, and $U_0 = 19$ kv. Using Eq. (30), the radius corresponding to 400 coul/kg particles was found to be 215 Å. Using $\epsilon_0^2 = \frac{2}{27}$ and $\theta = 30°$, the specific charge is found from Eq. (32) to be $q/m = 384$ coul/kg.

These calculated results are presented not to purport the validity of Eqs. (32) and (33), but rather to indicate their plausibility. These results indicate, however, that space charge has a greater effect, i.e., has a greater reduction in the electric field ratio, for high specific charges than for low values.

V. Conclusions

The results indicate the complex nature involved in the electrical atomization of liquids. However, it appears that a dynamic equilibrium is established by which the space charge, evaporation rate, voltage drop across the liquid meniscus, and surface energy extremum of the dispersed system of
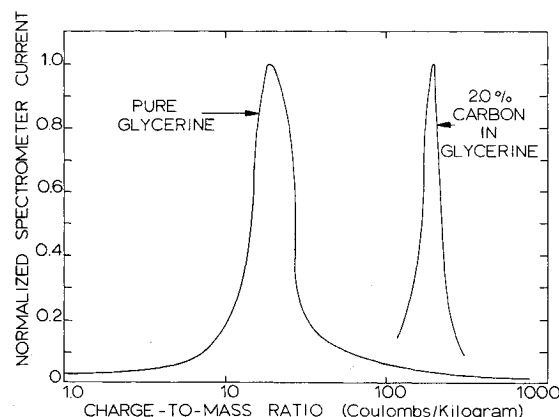
Fig. 9 Current distributions of specific charge for glycerine and a glycerine solution containing carbon particles. The operating conditions for each case were: accelerating voltage = 17 kv d.c., reservoir pressure = 16 cm of fluid, chamber pressure = 8×10^{-6} mm Hg.

particles act simultaneously to establish the radius and specific charge of the emitted particles. Further, the specific charge of electrically emitted particles can be enhanced considerably by placing colloidal particles in the solution to be dispersed.

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